MATH 155 - Chapter 8.3 - Trigonometric Integrals

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1. **Evaluating** $\int \sin^m x \cos^n x \, dx$**:**

   Case 1. n=odd: Save one cosine factor and use $\cos^2 x = 1 - \sin^2 x$. Let $u = \sin x$.

   Case 2. m=odd: Save one sine factor and use $\sin^2 x = 1 - \cos^2 x$. Let $u = \cos x$.

   Case 3. n=even, m=even: Use half-angle identities, and other identities.

   $$\sin^2 x = \frac{1}{2}(1 - \cos 2x), \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x), \qquad \frac{1}{2}\sin 2x = \sin x \cos x$$

   Case 4. n=odd and m=odd: Cannot use these formulas give above. Try something else!

2. **Evaluating** $\int \tan^m x \sec^n x \, dx$**:**

   Case 1. n=even: Save one factor of $\sec^2 x$ and use $\sec^2 x = 1 + \tan^2 x$. Let $u = \tan x$.

   Case 2. m=odd: Save one factor of $\sec x \tan x$ and use $\tan^2 x = \sec^2 x - 1$. Let $u = \sec x$.

   Case 3. Other than case 1 and 2: Cannot apply the formulas given above. Try something else.

3. **Useful Formulas:**

   1. $\int \tan x \, dx = \ln|\sec x| + c$
   2. $\int \sec x \, dx = \ln|\sec x + \tan x| + c$
   3. $\int \csc x \, dx = \ln|\csc x - \cot x| + c$
   4. $\sin(x + y) = \sin x \cos y + \cos x \sin y$
   5. $\sin(x - y) = \sin x \cos y - \cos x \sin y$
   6. $\cos(x + y) = \cos x \cos y - \sin x \sin y$
   7. $\cos(x - y) = \cos x \cos y + \sin x \sin y$